STRAIN GAGE ANGULAR ACCELERATION SENSOR

FACILITATES PRECISE CONTROL OF VIBRATION TABLE

Howard S. Havlicsek
(Acutronic USA, Inc.)

Presented
14 September 1999

Symposium Gyro Technology 1999
Stuttgart, Germany

STRAIN GAGE ANGULAR ACCELERATION SENSOR FACILITATES PRECISE CONTROL OF VIBRATION TABLE

Howard S. Havlicsek
(Acutronic USA, Inc.)

Abstract

Key words: Angular Accelerometer, Strain Gage, Vibration Table

Angular vibration tables are often used for product development and on the production floor to test and characterize inertial sensors. The fidelity, stability and frequency response of synthesized vibratory motion are significant factors that can limit the ability to fully test inertial products. This paper offers an approach to improving the performance of angular vibration motion simulators by recognizing those factors that limit performance and then employing a sensing technique that overcomes one dominant deficiency. This paper discusses the analysis, design, and testing of an angular acceleration sensor and summarizes operation and performance in the context of an Acutronic rotary vibration table. The sensor can be incorporated into systems of various configurations depending on mechanical constraints and performance goals. Many of the classical problems with conventional angular accelerometers are minimized due to the symmetry and inherent simplicity of this angular sensor and dynamic sensor response is completely eliminated. Theoretical and empirical data are presented to substantiate the analytical models.

Introduction

Angular vibration synthesis has traditionally been implemented on conventional rate tables using either an angular accelerometer or multiple tangentially mounted linear accelerometers for feedback sensing. System performance is often compromised by the phase shift (bandwidth) of the feedback sensor, the shaft compliance between the motor and the table top, and/or off axis alignment sensitivities. Controller compensation includes notching mechanical resonances and gain profiling filters all of which add more phase shift and contribute to reduced closed loop bandwidth of the motion system.

This paper defines a unique solution for an angular vibration table which uses a strain gage angular sensor to measure the angular acceleration of the table top. The sensor consists of a member attached at opposite ends of the output shaft such that it experiences the windup of the shaft whenever torque is transferred from the motor to the table top. The strain gage sensor measures the shaft output torque, which is proportional to the free body acceleration of the table top and load. Acceleration measured in this manner has no performance limiting dynamics, much the same way that a perfect angular accelerometer mounted directly on the table top would sense acceleration with infinite bandwidth and zero phase shift. In this configuration, the entire vibration table (including the UUT) becomes a closed loop angular accelerometer.

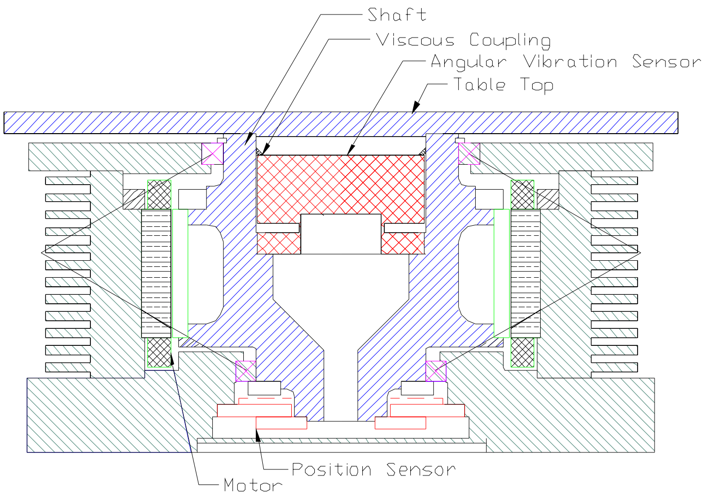
The advantage of this approach is that the additional dynamic error of a realistic accelerometer required in a conventional vibration table is eliminated and total open loop phase shift is significantly less due to the lower order of the plant/sensor dynamics. The closed acceleration loop can then be adjusted for higher bandwidth and less closed loop phase shift. The controller implements a fairly conventional high bandwidth inner acceleration loop and a low bandwidth outer position loop provides null drift stabilization. In practice it is possible to achieve acceleration bandwidths that are beyond the shaft resonant frequency.

The frequency response of a conventional vibration table is historically limited to 500 Hz using available acceleration sensors. A classical table design has demonstrated bandwidth in excess of 1000 Hz with less than 3 dB deviation in amplitude response by incorporating a strain gage acceleration sensor. When frequency response requirements are pushed to the limit many factors come into the picture that are normally ignored. Factors such as base reaction, shaft resonance, UUT/fixture mounting, motor reactance, external disturbance, power amp bandwidth, or friction, could degrade the performance of a system. The design of a vibration table must consider these issues to ensure that their combined effect can be managed.

Design

1. General description of the angular sensor in the context of an Acutronic vibration table.

The Acutronic series 100/AV angular vibration table was developed under contract for the purpose of production testing and quality control of motion sensitive sensors. The performance goal of the design was to achieve a peak angular acceleration of 50-60,000°/sec² and to synthesize sinusoidal stimulation of the UUT up to a frequency of 1KHz. Figure-1 is a mechanical assembly of the table and identifies the principal components. The motor is a 3 phase brushless design which is mounted in a housing that is forced air cooled. The bearing system consists of high precision angular contact ball bearings that have been customized for low friction and vibratory motion. A multi-pole position sensor is mounted at the bottom end of the shaft and is used to close a caging loop around the inner acceleration loop and supports position control in the 0.1 arc-min range. The table top is mounted to a relatively large diameter hollow shaft which ensures a shaft resonance greater than 3.5 KHz with the nominal specified load of 0.1 Kgm². Finally, the acceleration sensor is mounted in the center of the shaft near the table top. This is also the ideal location for a commercial accelerometer as required in a traditional vibration table configuration.



Angular Vibration Table Figure 1

2. Commercial Accelerometers

Let us first consider the case of a rotary vibration table that is constructed using a commercial angular accelerometer for closing a high fidelity acceleration loop. Angular accelerometers are designed and packaged in various styles and are engineered to satisfy a variety of sensing applications. The preferred configuration for a vibration table sensor is a single housed unit with integrated electronics and suitable performance specifications. The sensor package must be small and closely coupled to the table top and UUT. The axis of the sensor should be mounted concentric to the table axis to minimize off axis sensitivity; therefore, the ideal location for the angular sensor is on the underside of the table top in the hollow of the driving shaft.

A classical alternative sensing technique uses two linear accelerometers tangentially mounted at the outer limits of the table top. This approach involves meticulous electrical and mechanical alignments to ensure that the two units work as one. The linear accelerometers must be very high quality with minimal sensitivity to the large radial accelerations experienced during peak rate operation. The two units must be identically scaled, compensated and aligned to avoid sensitivity to linear vibrations as in the case of multi-mode vibration systems.

A vibration table that uses a commercial sensor for feedback cannot perform any better than the measurement sensor. The dynamic response of the accelerometer is very critical since it represents the difference between the measured and the actual table acceleration. The specifications for candidate sensors generally reveal a fundamental tradeoff between amplitude response (flatness) and phase response (phase shift) for units of equivalent bandwidth. This is best understood by recognizing that an inertial accelerometer can be reduced to a model consisting of a mass or inertia and a suspension system or support element. The measure of acceleration is proportional to the torque or force acting on a reference body ($\alpha = T / J$) as it is constrained to follow the motion of the base. Two things become obvious; all accelerometers (open or closed loop) operate with finite response and they exhibit mechanical resonance.

A representative performance specification established for the purpose of this paper is based on specifications for various commercial angular accelerometers and on product experience. In a practical accelerometer design, the damping behavior critically effects the response of the of the sensor in the frequency range of interest. The required response is based on the specified closed loop bandwidth of $\omega = 1000Hz$ for the vibration table and on the angular accelerometer having a natural resonant mode at $\omega_n = 2000Hz$. In the analysis that follows, the accelerometer is modeled as a second order dynamic system. The text book transfer function ;

Frequency Response	DC to 1 KHz
Amplitude Ripple	± 3 dB
Phase Lag @1 KHz	± 5 Deg.
Undamped Natural Frequency	2 KHz Min.

$$G(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\left(2\zeta \frac{\omega}{\omega_n}\right)};$$

is solved for the amplitude and phase response;

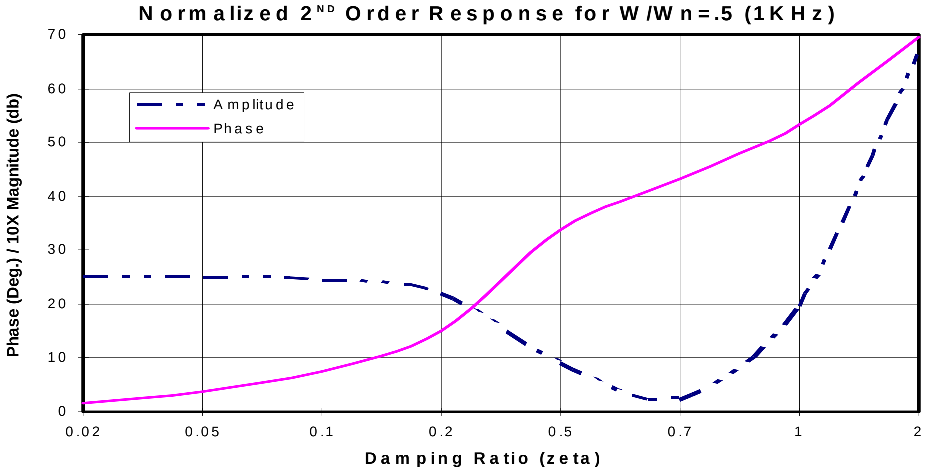
$$Amplitude = \left| 200LOG_{10} \sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2} \right|;$$

$$Phase = \left| \tan^{-1} \frac{2\zeta \frac{\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2}} \right|.$$

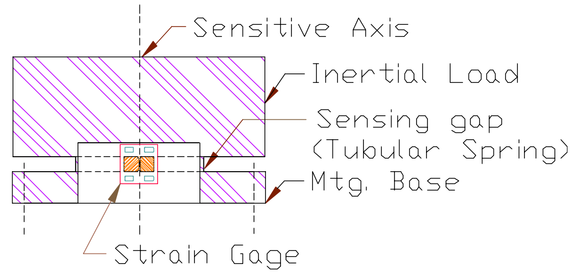
The chart below plots the normalized amplitude and phase responses as a function of damping ratio for the case where $\omega / \omega_n = 0.5$. Note that at the point where the amplitude response is a minimum ($\zeta = 0.7$) the phase shift is very large and unacceptable. The point where both the phase shift is below the 5° performance specification and the amplitude is less than 3 dB, occurs at $\zeta = 0.05$. This suggests that an under damped accelerometer response with 20 dB of peaking is needed to meet the feed back requirements over operating frequency range. This peaked response must be compensated in the control system to ensure closed loop stability.

Figure 2

3. Detailed description of angular sensor as an accelerometer.



The strain gage sensor was developed to function as an independent acceleration sensor and be used in conventional table designs; Figure 3. In this configuration the sensor offers limited advantage over conventional accelerometers because it has similar dynamic characteristics. It is described here because it embodies the principals of operation of the torque sensor.



Accelerometer/Torque Sensor
Figure 3

The sensor unit is constructed of 6061-T6 aluminum and is machined entirely from one cylindrical piece. The upper portion is the reference load inertia;

$$Jl = 0.0041\left(\left(Dl/2\right)^4 - \left(dl/2\right)^4\right)Thk * Density \quad \text{(in-lb-sec}^2\text{);}$$

where: *Dl* = outer diameter of inertial load
dl= inner diameter of the inertial load
Thk = thickness of inertial load

and is coupled to the lower mounting base by a tubular spring. This spring is machined into the sensing gap that separates the two portions of the sensor. The spring constant of the tubular spring is computed:

$$Ka = 0.0982G\left(D^4 - d^4\right)/l \quad \text{(in-lb/radian);}$$

where: *G* = modulus of elasticity;
D = outer diameter of tubular spring;
d = inner diameter of tubular spring;
l = length of tubular spring.

In the role of an accelerometer, the torque required to accelerate the inertial load is transferred from the base through the spring where the shear strain is concentrated.

The shear strain (γ) that results on the inside surface of the sensing gap is extremely uniform and linear:

$$\gamma = 5.093\frac{DT_m}{G\left(D^2 - d^2\right)} \quad \text{(PSI);}$$

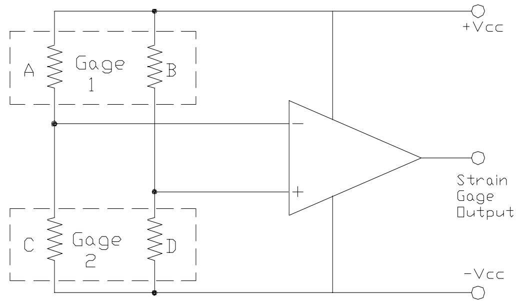
where: *Tm* = torque of acceleration.

Figure 3 shows an instrumentation grade strain gage bonded to the torsional spring. The gage is comprised of two elements on the same substrate which are oriented at ± 45° angles relative to the axis of sensitivity. A second gage is diametrically mounted to complete a full strain gage differential bridge. The alignment of the strain gages is important for canceling linear or translational coupling modes that are not along the sensitive axis.

A high quality differential amplifier is required to scale the signal to a useable level. The output voltage of the strain gage amplifier is $Vo = \gamma$ *GF*$G_{Amplifier}$ where:

GF = the gage factor for the strain gage.

Table 1 summarizes computed values of the equations above based on various geometrical parameters, operating conditions and materials. The amplifier gain computed for the second case is approximately 21,000 and results in a sensitivity of 200 rad/sec²/volt.



Accelerometer Analysis Table 1

	Material	Al	AL	SS	SS	SS	SS
	Density	0.1	0.1	0.283	0.283	0.283	0.283
Peak Accel.	**Ap (°/s^2)**	60000	60000	60000	60000	60000	60000
Twist Shaft	**D (in)**	1.203	1.2	1.01	0.38	1.5	2
	d (in)	1.183	1.18	1	0	1.49	1.99
	l (in)	0.15	0.15	0.25	0.25	0.25	0.25
Inertial Load	**Dl (in)**	2.48	2.48	2	2	3	2
	dl (in)	0	1.18	1	0	1.49	0
	Thk (in)	1	1	1.5	2	0.75	6
	G (PSI)	3800000	3800000	10600000	10600000	10600000	10600000
Inertia of Load	**Jl=**	0.000965	0.000915	0.001624	0.002309	0.004117	0.006928
Spring Constant	**k=**	337841.5	335299.3	169010.5	86791.87	556330.8	1322014
Resonant Frequency	**w=**	2978.541	3046.409	1623.763	975.7122	1850.048	2198.567
Peak torque	**Tm=**	1.010126	0.958353	1.700347	2.418271	4.311579	7.254814
Shear Stress	**tau=**	45.55886	43.44292	215.4089	224.4543	246.4409	232.669
Shear Strain	**gama=**	1.2E-05	1.14E-05	2.03E-05	2.12E-05	2.32E-05	2.19E-05
Double Bridge Voltage	**V10=**	239.7835	228.647	406.432	423.4987	464.9829	438.9982
Amplifier Gain (±5Vo)	**Gain=**	20852.15	21867.77	12302.18	11806.41	10753.08	11389.57

4. Detailed Description of the angular sensor as a torque sensor.

From the previous discussions it should be clear that the angular sensor is fundamentally a torque measuring device and in fact is very sensitive. The analysis of Table 1 indicates that the peak torque measured for full scale output is only 0.96 in-lb. The spring constant of the torque sensor is 335,000 in-lb/rad and the angular wind-up is computed as $\theta = \frac{T_M}{K_S} \cong 2.9$ µ-radians. From this perspective, the sensor is capable of measuring very small position displacements.

In contrast, the stiffness of the output shaft has been measured to be approximately $15x10^6$ ft-lb/rad and at peak acceleration transfers 35 ft-lb of torque. The wind-up of this shaft is approximately 2.3 µ-radians which is spread uniformly across a large portion of the shaft length.

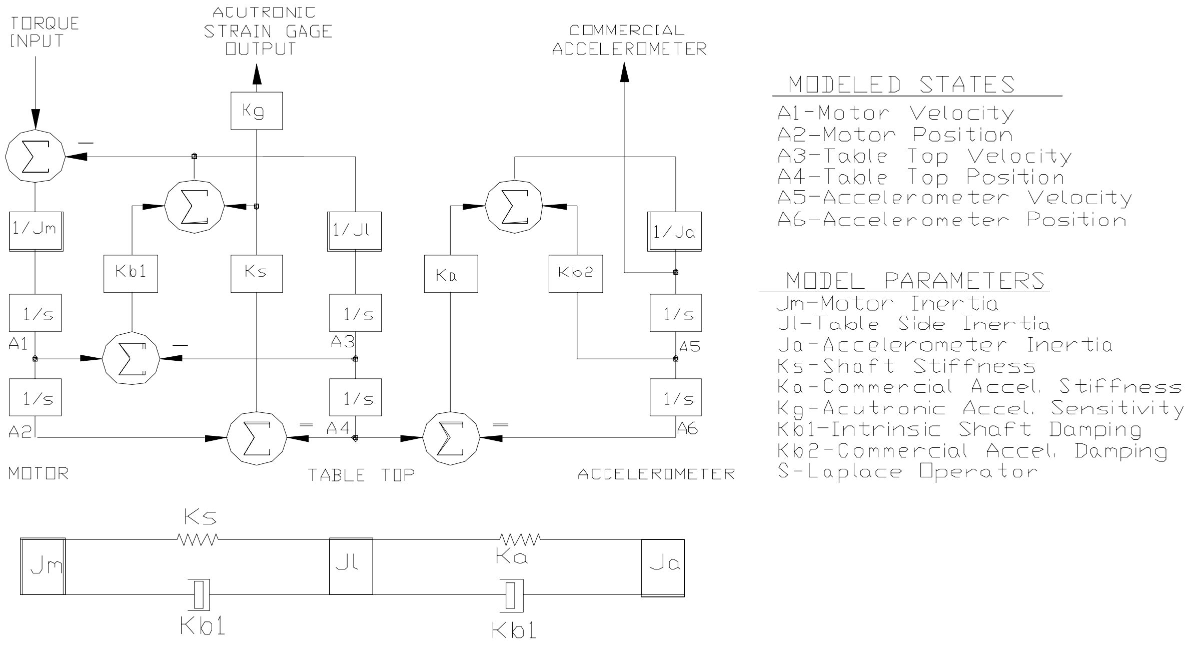
Referring back to Figure 1 of the vibration table, the angular sensor is securely mounted about half way down the table shaft and extends nearly to the table top where it is connected with a viscous coupling. The coupling is very stiff above 1 Hz, therefore the sensor appears to be very rigidly attached at both ends and spans about 2.5 inches of the shaft. The uniform twist of the shaft over this span is concentrated in the gap of the torque sensor. The ratio of 2.5 inch span to 0.15 inch sensing gap results in a mechanical gain of 16 to 1 or in other words it would require 16 times more electronic gain if the vibration table torque were directly instrumented with strain gages on the shaft.

The viscous coupling on one end of the shaft provides a floating mechanical connection which reduces the precision requirements on the mounting interfaces. It also prevents steady state wind-up of the sensor that could result in mounting and temperature sensitive offsets in the measurement.

Analysis

1. Plant Dynamic Model

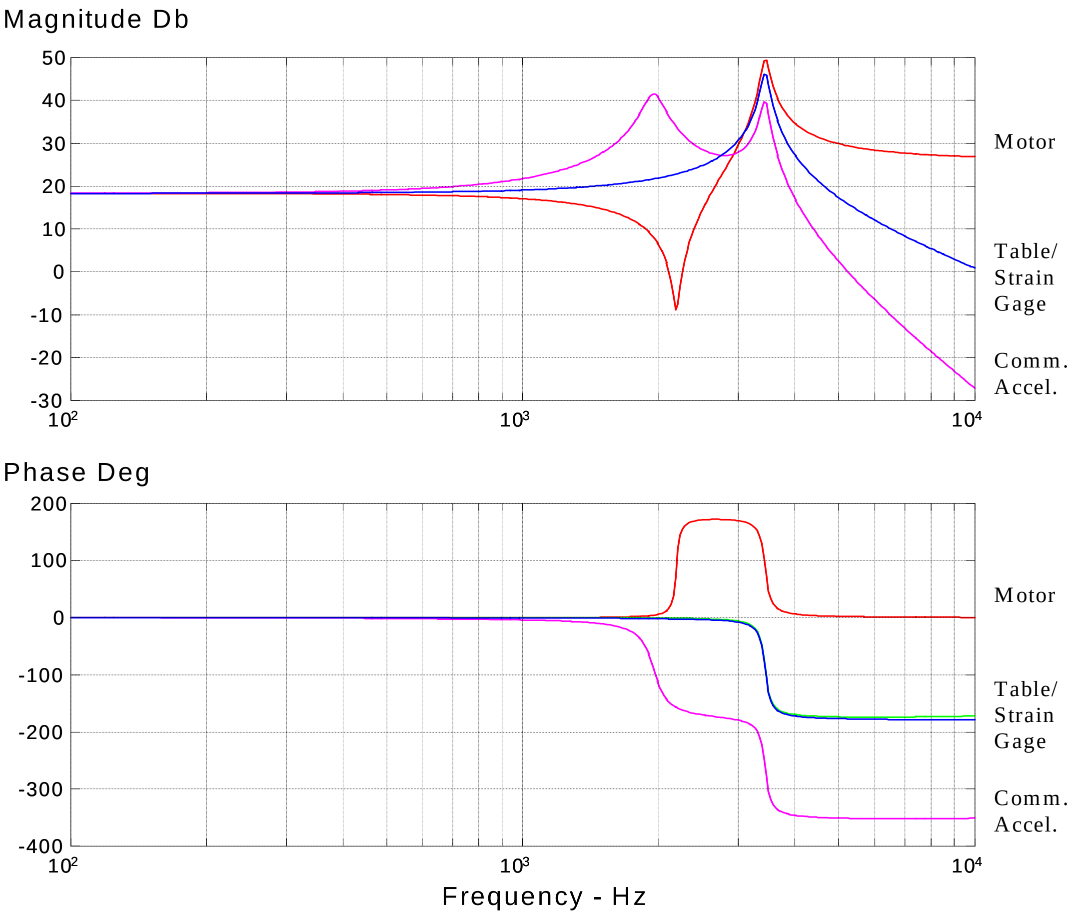
Figure 4 shows a lumped model of the plant which includes the dynamics of the shaft resonance and a dynamic model of a conventional accelerometer. The input to the plant model is the torque that is produced by the motor. Four outputs are plotted of which two are identified as the scaled strain gage output and the other is the commercial accelerometer output. The other two outputs are the acceleration of the motor side and table top side of the rotating assembly.



Plant Model Figure 4

```
**** MODEL PARAMETERS ****
Jm=.049;        % in-lb-sec^2   Motor Inertia
Kb1=25;         % in-lb/rad/sec Motor/Bearing Damping coeficient
Ks=13600000;    % in-lb/rad  Shaft Stiffness (lumped)
Jl=.072;        % in-lb-sec^2  Table Top Load Inertia
Ja=.0008;       % in-lb-sec^2  Accelerometer Equivalent Inertia
Ka=120000;      % in-lb/rad  Stiffness of Accelerometer sensing member
Kb2=10          % in-lb/rad/sec Accelerometer Electronic Damping coeficient
Kg=Ks/Jl;       % Constant that converts accelerometer strain to normalized units

**** MODEL DEFINITION ****
% ' A ' Matrix definition
a1=[-Kb1/Jm  -Ks/Jm   Kb1/Jm   Ks/Jm    0       0   ];
a2=[   1        0        0        0       0       0   ];
a3=[Kb1/Jl   Ks/Jl   -Kb1/Jl   -Ks/Jl    0       0   ];
a4=[   0        0        1        0       0       0   ];
a5=[   0        0        0      Ka/Ja  -Kb2/Ja  -Ka/Ja];
a6=[   0        0        0        0       1       0   ];
am=[a1;a2;a3;a4;a5;a6];

% ' B ' Matrix definition
bm=[1/Jm 0 0 0 0 0]';

% ' C ' Matrix definition
c1=[-Kb1/Jm  -Ks/Jm    Kb1/Jm      Ks/Jm       0          0   ];% Motor Acceleration
c2=[Kb1/Jl      Ks/Jl     -Kb1/Jl       -Ks/Jl        0          0   ]; % Table Top Acceleration
c3=[   0            0             0             Ka/Ja    -Kb2/Ja  -Ka/Ja];% Commercial Accelerometer
c4=[   0     Ks/Jl      0          -Ks/Jl        0       0  ];              % Strain Gage output
cm=[c1;c2;c3;c4];

% ' D ' Matrix definition
dm=[1/Jm;0;0;0];
```

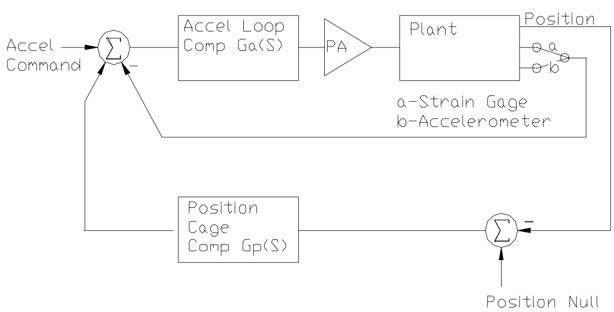
Modeled Acceleration Plant   Figure5



The motor side inertia (*Jm*) of the vibration table exhibits an anti-resonance at 2.1 KHz where it resonates with the table side inertia (*Jl*) through the shaft stiffness (*Ks*). At 3.3KHz *Jl* resonates with *Ks* and produces the characteristic second order peaked response. The accelerometer output includes both the dynamics of the table and the accelerometer which adds a resonant peak at 2 KHz. The strain gage output tracks the table top almost perfectly except at the high frequencies due to the effects of the natural viscous damping (*Kb1*). The accelerometer adds peaking and phase lag to the table response.

2. Control Strategy

The control architecture shown in Figure 6 represents the closed loop servo in simplified form. The inner loop consists of the dominant acceleration loop which needs to be appropriately compensated so that the performance goals are achieved. The outer position loop maintains the center position of the vibratory motion. This loop is compensated via Gp(S) and has a very low bandwidth to keep from disturbing the acceleration response at low frequencies. The outer position loop is not addressed .

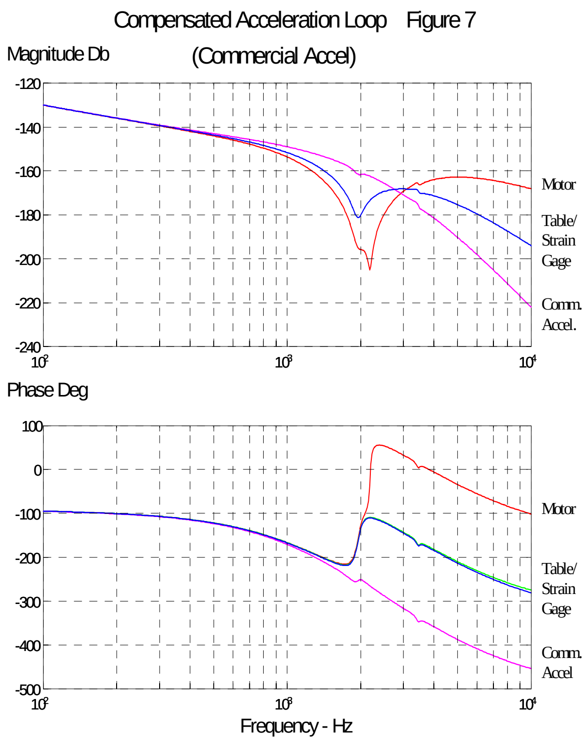


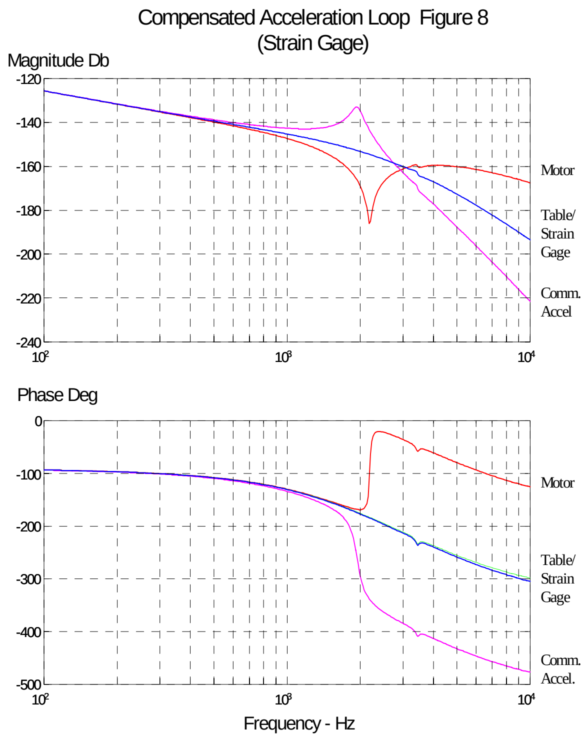
Elementary Servo Block Diagram   Figure 6

The plant transfer functions provide the key information to effectively compensate the closed acceleration loops. Closing acceleration loops around the commercial accelerometer and the strain gage feedback requires a variety of compensation filters which are represented by Ga(S) in Figure 6. A second order low pass filter is added to the plant dynamics to model the 2 KHz bandwidth of the power amplifier.

The following table summarizes the compensation filters that are used for the two cases. The compensated open loop transfer functions are plotted in Figure 7 and Figure 8:

Compensation Filter	Commercial Accelerometer	Strain Gage Sensor
Power Amp (2 real poles)	Wpa=2000 Hz	Wpa=2000 Hz
Shaft Resonance (Notch Filter)	Wn1=3.3 KHz; ζ=.02	Wn1=3.3 KHz; ζ=.02
Accelerometer Resonance (Notch Filter)	Wn1=2.0 KHz; ζ=.04	(none)
Integrator With Lead Break	Wl=600 Hz; Gain=142 dB	Wl=1000 Hz; Gain=143 dB

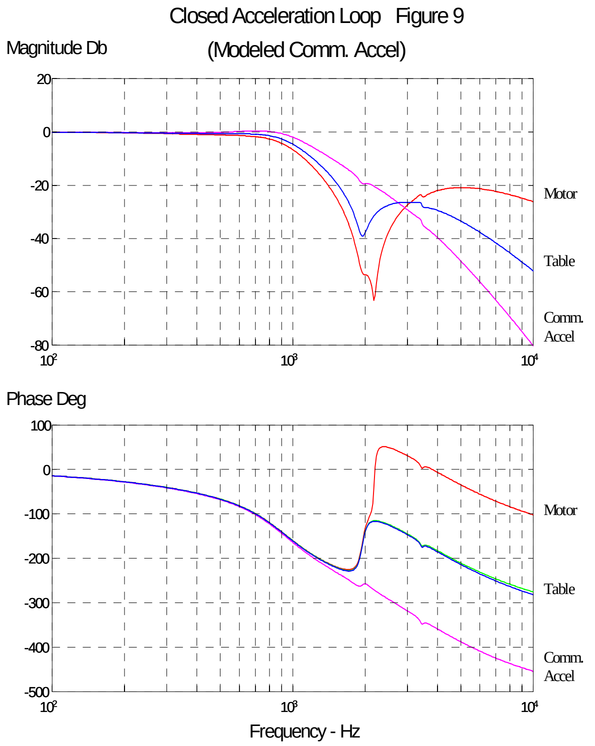


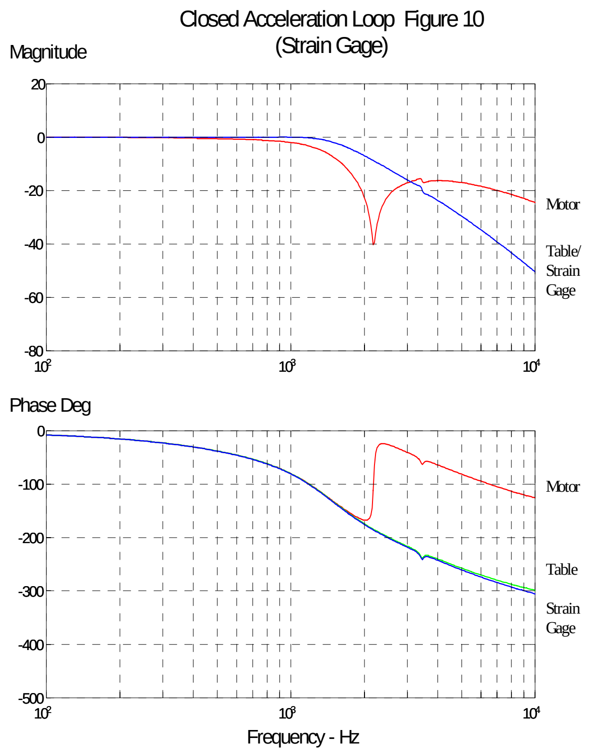


The compensation filters have been adjusted so that the phase margin at crossover is greater than 50° to ensure that the closed loop peaking be kept to a minimum. The crossover frequency for the accelerometer feedback loop is considerably less than the strain gage loop as expected.

The closed loop transfer functions are plotted in Figure 9 and Figure 10. The closed loop performance is summarized as follows:

Performance	Commercial Accelerometer	Strain Gage Sensor
Bandwidth (90° phase)	600 Hz	1050 Hz
Bandwidth (-3 dB)	1100 Hz	1700 Hz
Ripple (to 1000Hz)	+0, -3 dB	+0, -0.5
Sensor error (@ 1KKz)	-2 dB	0





Conclusions

Vibration table performance can be significantly improved by eliminating excess phase in each element of the servo system. The measurement sensor is only one of many places where dynamic errors can reduce performance, however, the sensor is the last word in reporting the true motion of the table top test surface. If the measurement sensor is in error then no amount of closed loop bandwidth will guarantee the fidelity of vibratory motion.

The use of a strain gage sensor to measure the accelerating torque acting on the free body load of an axis offers an alternative method to measure and control the acceleration of a vibration table. This technique provides an extremely linear, stable and accurate measurement with negligible added dynamics.

References

1. T. Kailath, Linear Systems,Prentice-Hall, Inc., Englewood Cliffs, N.J. ,1980

2. J. Schwartzenbach and K.F. Gill, Halsted Press, John Wiley & Sons, Inc., New York, 1978

3. C. Moler, J. Little & S. Bangert, PC-Matlab, Math Works, Inc. Sherborn, Ma, 1987