Making Flight Motion Tables Invisible

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\textbf{ABSTRACT}
Flight tables can add unwanted dynamics with increased phase lag and gain attenuation to the Hardware-In-The-Loop (HWIL) simulation. By making flight tables “invisible” we reduce the effects of these unwanted dynamics on the simulation giving the simulation engineer a much clearer picture of the test unit’s capabilities. Past methods\textsuperscript{1} relied on clever servo techniques to reduce these effects. In this paper we look at the mechanical aspects of the flight table; in particular, we study the effects of using composite materials in the fabrication of the flight table gimbals. The study shows that the use of composite gimbals significantly increases the invisibility of the flight table with the potential added benefit of reduced cost.

\textbf{Keywords:} composite gimbals, flight tables, HWIL, low cost

\textbf{1. INTRODUCTION}
We present a design study and servo analysis of a three-axis flight table constructed mainly of composite gimbals and shafts and supporting a typical payload. A sketch of the flight table is shown in figure 1-1.
Current state-of-the-art, HWIL simulation facilities employ either one or two flight motion tables. A typical configuration uses a three-axis missile motion table and a two-axis target motion table. The flight motion tables respond to commands from the missile guidance system and target motion generator to simulate the missile attitude (roll, pitch and yaw) and the line-of-sight (azimuth and elevation) between the missile and target during the HWIL engagement simulation. It is very important that the motion tables appear “invisible” to the HWIL simulation; by this we mean that the flight tables must produce negligible dynamic and static errors during the simulation as “seen” in the test results. The invisible flight motion table allows the HWIL simulation engineers to evaluate the missile system/target engagement capabilities without the added complexity of compensating for the flight motion table errors. This situation has become even more acute with the development of more complex modern guided weapons systems.

Figure 1-2 is a photograph of a current model flight table manufactured by Acutronic.

![Current Acutronic Three-Axis Flight Table Using Aluminum Gimbals and Hydraulic Actuators](image_url)

These conventional flight tables are designed with aluminum or magnesium gimbals and can achieve axes servo bandwidths of about 30 Hz. Additionally, the relatively heavy gimbals and high rotating inertias require the use of hydraulic actuators to produce the typical 20,000 deg/sec² angular acceleration.

This design study compares a current state-of-the-art, aluminum-based, flight motion table with the new, composite gimbal design. Comparisons are made in the areas of:
- mechanical layout and component selection
- structural mode locations,
- servo bandwidth, and
- dynamic performance.

The new approach offers the desired invisibility feature at a potentially lower cost.

2. BACKGROUND AND MOTIVATION

We can reproduce a simple illustration of the effect that the flight table dynamics have on the overall HWIL simulation\(^1\). Consider the two-dimensional air-to-ground engagement geometry shown in figure 2-1.
The LOS angle, $\lambda$, is the angle between the inertial vertical and the range vector, $R$. Both $\lambda$ and $R$ vary as the engagement plays out. The relationship between $\lambda$ and $R$, for small $\lambda$, is simply:

$$\sin(\lambda) = \frac{X_t - X_m}{|R|} \approx \lambda$$

Figure 2-1 also shows the body orientation of the missile during the engagement with the target. We can see in the figure how the missile airframe orientation relates to the overall engagement geometry variables. We assume that a target seeker is onboard the missile that generates a bore-sight error angle, $\theta_B$, between the missile airframe and the target. We can measure the airframe orientation angle, $\theta_A$, with a strapped down gyro on the missile body or as a part of the seeker. We write the relationship for $\theta_B$, $\theta_A$ and $\lambda$ as:

$$\lambda = \theta_A + \theta_B$$

Figure 2-2 is a simplified block diagram of the two-dimensional engagement problem showing the overall missile-to-target engagement geometry and the missile body orientation. The figure contains a missile seeker and autopilot and uses a proportional navigation tracking algorithm.
The variables shown in figures 2-1 and 2-2 are:

1. \( \lambda \) ≡ Line-of-Site (LOS) angle, from missile to target
2. \( X_m \) ≡ missile position
3. \( X_t \) ≡ target position
4. \( R \) ≡ range vector from missile to target
5. \( V_m \) ≡ missile velocity vector
6. \( V_t \) ≡ target velocity vector
7. \( \hat{\lambda} \) ≡ simulated LOS Angle
8. \( \dot{\lambda} \) ≡ LOS angular rate estimate
9. \( \dot{\theta}_A \) ≡ simulated airframe orientation angle
10. \( \theta_B \) ≡ simulated seeker boresight error angle
11. \( V_c \) ≡ closing velocity
12. \( \eta_c \) ≡ normal acceleration command to missile

The hardware elements in the figure are shaded and consist of:

- target position table
- missile flight table
- missile seeker and tracker
- body rate gyro

The scenario defined by figure 2-2 can also be played out totally with a simulation computer. We can run the simulation in either real time or non-real time to compute the “miss distance”, \( (X_t - X_m) \), for various models of the target, missile, seeker, etc. We increase the realism of the simulation by adding some of the flight hardware to the simulation; e.g. the missile seeker and the body rate gyro. Adding the flight hardware also requires the three-axis missile flight table to simulate the missile body orientation and the two-axis target position table to simulate the missile-to-target LOS.

The figure shows the significant effect that the target position table and missile flight table will have on the overall fidelity of the simulation and demonstrates the need for transparency. Any motion table dynamic lag or non-linearity will compromise the simulation. From a “large signal” standpoint, the flight table must produce the high accelerations and rates demanded by the scenario. From a small signal, input/output transfer function standpoint the flight table must have unity gain and zero degrees phase lag for all frequencies to be perfectly transparent. However, we will settle for minimum phase lag and gain attenuation over the frequency band of interest. Usually this is 10 Hz and below for the three-axis table in the inner autopilot loop and 1 Hz and below for the two-axis table in the outer missile-target engagement loop. However, with the more complex missile system, these two parameters could double or triple.
The problem of designing a flight table that is “invisible” to the HWIL simulation becomes a classical trade-off study. The table must produce high accelerations and low servo phase lags. The efficient way to produce the high acceleration is to reduce the axis inertia. Reducing inertia can result in lightweight but often compliant gimbals. The compliant gimbal results in a low torsional resonance that has the effect of reducing the axis servo bandwidth. Our basic problem is to develop a lightweight, low inertia mechanical system (e.g. gimbal, shaft, bearings, etc) that is also extremely stiff. Our solution is to design a three-axis flight table using mostly composite gimbals and shafts in a relatively compact, very stiff, conventional configuration.

The torsional resonance of the flight table gimbal may be approximated as:

\[ \omega = \sqrt{\frac{K}{J}} \] (2.1)

\( K \) is the gimbal torsional stiffness in \( \frac{\text{ft lbs}}{\text{rad}} \) that is directly related to both the shear and bending material moduli of elasticity. By appropriate gimbal and shaft interface design, we can make this dependence more directly onto the larger bending modulus, \( E \) in \( 10^6 \text{ psi} \). \( J \) is the angular moment of inertia in \( \text{ft} - \text{lb} - \text{sec}^2 \) that is directly related to the gimbal geometry and material density, \( \rho \) in \( \frac{\text{lbs}}{\text{in}^3} \). Generally, the torsional resonance is proportional to the modulus-to-density ratio, \( \frac{E}{\rho} \), i.e.:

\[ \omega = \sqrt{\frac{K}{J}} \left[ \frac{E}{\rho} \right]^{\frac{1}{2}} \]

Table 2.1 presents a comparison of some of the most relevant material properties for aluminum, magnesium and carbon composites.

<table>
<thead>
<tr>
<th>Property</th>
<th>( \rho ), density, lbs/in(^3)</th>
<th>( E ), E-Modulus, 10^6psi</th>
<th>( E/\rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum(^5)</td>
<td>0.1</td>
<td>10</td>
<td>100</td>
</tr>
<tr>
<td>Magnesium(^5)</td>
<td>0.07</td>
<td>6.5</td>
<td>93</td>
</tr>
<tr>
<td>Carbon Composites(^4)</td>
<td>0.06</td>
<td>21</td>
<td>350</td>
</tr>
</tbody>
</table>

So, from a material property alone, the torsional resonant frequency for composites should be \( \sqrt{3.5} \) times that of aluminum or magnesium.

We factor in a geometric relationship by comparing simple gimbal configurations. Figure 2-3 is a sketch showing a the cylindrical geometry of the composite gimbal and a simple beam geometry of the aluminum gimbal.
Using the material densities and these simple geometries we can approximate the ratio of the composite, cylindrical gimbal inertia to the aluminum, rectangular gimbal. Assuming equal volumes, the moment of inertia ratio is:

\[ \frac{J_C}{J_A} \approx \frac{1}{2} \left( \frac{\rho_C}{\rho_A} \right) \]

From Table 2-1, the modulus of elasticity, \( E \), of wound carbon fiber composites is a factor of two and three to that of aluminum and magnesium respectively. So the ratio of the composite to the aluminum stiffness is approximately:

\[ \frac{K_C}{K_A} \approx \frac{E_C}{E_A} \approx 2 \]

Combining these ratios in equation (2.1) results in:

\[ \frac{\omega_C}{\omega_A} \approx \sqrt{\left( \frac{K_C}{K_A} \right) \left( \frac{J_A}{J_C} \right)} \approx \sqrt{\left( \frac{E_C}{E_A} \right) \left( \frac{2 \rho_A}{\rho_C} \right)} \approx 2 \sqrt{\frac{\rho_A}{\rho_C}} = 2 \sqrt{1.7} = 2.6 \]  

This indicates that the composite, cylindrical design will produce a torsional resonance about a factor of two of the aluminum, rectangular gimbal.

Another significant advantage of the cylindrical gimbal design over the more conventional rectangular design is the elimination of the weak bending locations at the corners. These relatively weak corner locations result in lower stiffness in torsion and reduced torsional resonances. The cylindrical design has no local weak areas.

The cylindrical shapes of the flight table gimbals and the high specific modulus not only maximize torsional stiffness but also produce a design that is relatively easy to manufacture using standard, commercially available carbon fiber tubing. The design yields a relatively low cost flight table with the capability of producing lowest torsional modes and servo axis bandwidths a factor of almost three to that of the aluminum (or magnesium) flight table.

HWIL test and simulation laboratory combines physical effect simulators with computers operating in real time. The physical effect simulators generally consist of:

- three-axis table that simulates the roll, pitch and yaw of the missile motion
- two-axis table that simulates the azimuth and elevation of the target Line-of-Sight(LOS) angle, and
- target scene generator that simulates the target image in the required wavelengths.

The three and two axes flight tables in the HWIL laboratory must appear “transparent” with respect to the dynamic behavior of the missile-target engagement scenario in order to produce a high fidelity simulation. The motion simulators must produce motion that has minimal phase lag and gain attenuation over the frequency band of interest and also exhibit minimal non-linear motion for large signal commands.
3. MECHANICAL DESIGN CONCEPT AND ANALYSIS

Our technical approach uses previous efforts funded by the Air Force; specifically, the Improved Three Axis Test Table (ITATT) program\(^2\). Of primary interest was the selection of carbon composites for the gimbal material in order to minimize inertia and maximize stiffness. Additional ITATT work\(^3\) describes the mechanical design, structural analysis and fabrication of the ITATT inner gimbal. The inner gimbal was constructed of graphite/epoxy with Invar fittings in a rectangular configuration. The gimbal weighed about 70 pounds and was about three feet wide by three feet high by one foot deep by 6 inches thick. The harmonic analysis of the gimbal showed a lowest structural torsional mode at 1,350 Hz. The gimbal was eventually installed in a single axis proof-of-principle demonstration of ITATT.

3.1 Mechanical Design Concept
Figures 3-1a and 3-1b show the front and side views of the composite three-axis flight table design.

![Composite Flight Table Front View Showing the Cylindrical Geometry and Overall Dimensions](image1)

![Composite Flight Table Side View Showing the Test Package and Field of View](image2)
The base supports the outer (pitch) axis; which, in turn, supports the middle (yaw) and inner (roll) axes. The pitch and yaw axes will be limited in rotation and the roll axis will be continuous. Each axis assembly will include gimbal structures, drive shafts, bearings, torque motors, and an Inductosyn feedback sensor. Only the inner axis will have slip rings in order to supply signals to the payload under continuous roll axis motion. The torque motors will be brushless AC motors with permanent magnet rotor rings and stationary armature coils to minimize rotating inertia.

We construct the outer and the middle axis’ gimbals of cylindrical carbon composite tubing in a layered “sandwich” design. There are at least two U.S. companies that fabricate carbon fiber composite tubing; these are Amalgma Composites Inc, www.amalgacomposites.com, and Polygon Co., www.polylube.com. Amalgma has the capability to build composite tubes up to 42 inches in diameter and Polygon has the capacity to build tubes up to 30 inches in diameter.

The inner axis consists of simple, composite tubular structures attached to the middle gimbal back plate. The motor and bearing set are attached to the tubular composite shaft and the middle gimbal back plate. We use a collett mounting arrangement for the test package. Using composites minimizes weight and increase the overall stiffness.

The middle gimbal consists of two concentric, cylindrical % inch thick tubes separated by 4 ½ inches by 12 inches long and weighing approximately 32 pounds. The outermost tube is 30 inches in diameter. The middle axis’ shafts are composite tubes 9 inches in diameter with a wall thickness of 1 inch. We attach the middle axis motor armature coils and bearings to the middle gimbal “stationary” shafts with the permanent magnet ring attached to the rotating gimbal shaft interface plates. This reduces the rotating inertia for the middle gimbal to about 3 ft-lb-sec² without the test package. Two 500 ft-lb torque motors, each weighing approximately 50 pounds, provide the comparable 20,000 deg/sec² of angular acceleration to the middle axis. Although the motor weight and location will not add appreciably to the middle axis rotating inertia, they will add to the outer axis inertia.

The outer gimbal is of similar construction to that of the middle gimbal and consists of two concentric, cylindrical % inch thick tubes separated by 3 ¼ inches thick separated by 3 ¼ inches. The outermost tube is 42 inches in diameter, 12 inches long and weighs approximately 87 pounds. The outer axis’ shafts are composite tubes 11 inches in diameter with a wall thickness 1 ½ inches. We invert the motor configuration for the outer axis by placing the permanent magnet ring on the rotating shaft with the armature coils located within the base structure. The total rotating inertia of the outer axis including the middle axis torque motors will be approximately 20 ft-lb-sec².

Figure 3-1 shows the axes travel range of approximately ±45° with an instantaneous Field-of-View of approximately ±45°. The approximate, overall dimensions for the entire flight table are 6 feet wide, 4 feet high and 3 feet deep.

The construction of the gimbal tubes and the mechanical interface between the shafts and the gimbal are critical design considerations in order to spread loads and keep the structure rigid. Figure 3-2 shows these concepts.
Figure 3-2  Design Concept for Tubular Sandwich Gimbal and Shaft Mechanical Interface for both the Outer and Middle Axes

The inner and outer gimbal tubes are held together with a combination of circular front and back face plates and tie rods. The shaft interface is bolted to the faceplates, avoiding penetrations of the outer gimbal for the outer shaft connection. All the interface materials are aluminum or steel; and all boltments are metal to metal. The pitch and yaw composite drive shaft tubes will be either bolted or collett mounted to their respective shaft interface plates. This shaft interface design concept converts the potential gimbal shear loads from the shaft torque motor to gimbal bending loads. By eliminating the shear loads on the composite gimbal and replacing them with bending loads we can correctly assume that the gimbal material modulus will be more like the larger E-modulus than the smaller shear modulus. The middle axis design concept is similar to the outer axis; a hole through the outer gimbal will allow the attachment of the middle axis drive shaft. We will fill the space between the inner and outer tubes with a rigid polyurethane foam that will provide additional damping to the gimbals.

3.2 Mechanical Structure Analysis
We performed extensive Finite Element Analyses of the composite flight table using the ANSYS FEA tool. Our aim was to predict the lowest torsional mode for the outer and middle axes. Figure 3-3 shows our Finite Element Model mesh for the composite flight table.
Figure 3-3  
FEM Mesh used for the Composite Gimbal System

Figure 3-4 shows some of the FEA results without the test package; specifically the lowest torsional modes for the outer and middle axes:

Figure 3-4  
FEA Results Showing the Outer and Middle Axes Lowest Torsional Resonances of 363Hz and 835 Hz

With the base structure fixed, the FEA predicted the lowest torsional modes for the outer and middle axes at 363 Hz and 835 Hz respectively. In a final design configuration, we could construct the base from cast iron to create a massive, stable platform. The harmonic effect of this base design on the outer axis frequency response will be low residue dipole (back resonance from the base) that could be completely ignored.
We can compare the mechanical design and FEA results of the composite flight table with a conventional, aluminum gimbal based, design. Figure 3-5 is an FEM of a conventional flight table that Acutronic designed and built several years ago. The conventional flight table is comparable in size to the composite flight table.

Table 3-1 compares some of the overall physical characteristics of the conventional table design with the composite table.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Axis Travel (Deg)</th>
<th>Instantaneous Field of View (Deg)</th>
<th>Test Package Diameter (inches)</th>
<th>Test Package Aft</th>
<th>Outer Gimbal Dimension (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Composite Table</td>
<td>±45</td>
<td>±45</td>
<td>7</td>
<td>29</td>
<td>42 (Dia) X12(long)</td>
</tr>
<tr>
<td>Conventional Aluminum Based Table</td>
<td>±45</td>
<td>±45</td>
<td>7</td>
<td>26</td>
<td>48X42X12</td>
</tr>
</tbody>
</table>

Table 3-2 compares the results of the conventional table design and analysis with the composite table for the outer axis only. All data was produced without the test package installed.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Axis Inertia (Ft-Lb-sec²)</th>
<th>Axis Peak Torque Required for 20,000 deg/sec² (ft-lbs)</th>
<th>Axis Lowest Torsional Resonance (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon Composite Table</td>
<td>20</td>
<td>7,000</td>
<td>363</td>
</tr>
<tr>
<td>Conventional Aluminum Based Table</td>
<td>100</td>
<td>35,000</td>
<td>150</td>
</tr>
</tbody>
</table>

Note: Inertias and resonance data are based on no payload

Table 3-2 shows a significant advantage of the composite flight table over the conventional aluminum flight table:
1. The lower inertia results in 5 times less axis peak torque for the same angular acceleration, and 
2. The higher torsional resonance will result in significantly higher axis servo bandwidth.

4. SERVO DESIGN CONCEPT AND ANALYSIS

We can demonstrate the benefit of the high torsional resonance by completing a typical design of the outer axis servo system. We use a control configuration consisting of a rate loop within a position loop with feed-forward commands. The encoding system will use a multi-speed Inductosyn (fine channel) and a single speed resolver (coarse channel) to produce an absolute position sensor measurement with a state observer to estimate and predict the rate and acceleration states. Figure 4-1 shows this conventional control system architecture.

Figure 4-1 Typical Control System Architecture Showing Interconnection of the Servo System Elements and the Physical Plant for

The figure shows the State Observer producing the rigid body states, the actuator motor current feedback servo sub-loop, the rate and position controller, and the state vector command generator with feed-forward commands. Our first step is to develop the mathematical model for the flexible plant dynamics based on the FEA results above.

4.1 System Model

The primary constraint to extending the small signal frequency responses of the axis’ servo systems is the frequency domain location of the axis’ torsional resonant modes. Typically, we would like the axis bandwidths to be an octave below the lowest torsional resonance. Our FEA results presented in section 2 above for the outer and middle axes, using cylindrical symmetry and composite materials, show lowest torsional resonance of 363 and 835 Hz respectively. Figure 4-2 is a lumped, three-body, dynamic representation of the rotating gimbal (typical of yaw and pitch axes) driven from both sides.

Figures 4-2 Three-Body Lumped Model Representation of the Flexible Outer Axis Gimbal Driven from Both Sides and the Rigid Base Structure Showing the Shaft, Gimbal and Base Bodies
We call the two moving bodies in the model the “shaft” and the “gimbal”. The “shaft” body, however, also includes part of the flexible gimbal since it moves relative to the rest of the gimbal in this lumped approximation. We assume that the base is much stiffer and more massive than the gimbal and is stationary. The model representation also assumes symmetry of the shaft compliances, shaft inertias and applied torques. We define the parameters of the model shown in figure 4-2 as

\[ J_b : \text{Base Inertia} \]
\[ J_g : \text{Gimbal Inertia} \]
\[ J_s : \text{Shaft Inertia (including part of the gimbal)} \]
\[ K_g : \text{Gimbal Stiffness} \]
\[ \theta_g : \text{Gimbal Angle Relative to Base} \]
\[ \theta_s : \text{Shaft (Sensor) Angle Relative to Base} \]
\[ T_m : \text{Motor Torque} \]

The state-space model for the flexible plant, encompassing the gimbal and shaft and assuming that the axis position sensor and torque motor are co-located on the shaft body, is:

\[
\dot{x} = Ax + bu \\
y = cx
\]

where:

\[ y = \text{Sensor Output Angle, } \theta_s \]
\[ u = \text{Motor Torque, } T_m \]

and,

\[
X = \begin{bmatrix}
\dot{\theta}_g \\
\theta_g \\
\dot{\theta}_s \\
\theta_s
\end{bmatrix}, \quad B = \begin{bmatrix}
0 \\
0 \\
1/J_s \\
0
\end{bmatrix}, \quad C = 0 \quad 0 \quad 0 \quad 1
\]
\[
A = \begin{bmatrix}
0 & -K_g/J_g & 0 & K_g/J_g \\
1 & 0 & 0 & 0 \\
0 & K_g/J_s & 0 & -K_g/J_s \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

The transfer function relating the scalar output, \( y(s) \), to the input, \( u(s) \) is given by:

\[
\frac{y(s)}{u(s)} = c \quad sI - A^{-1} b
\]

This reduces to:

\[
\frac{y(s)}{u(s)} = \frac{1}{J_s s^2 + K_g/J_g} \quad \frac{1}{J_p s^2 + K_g/J_p}
\]

where

\[
J_p = \frac{J_g J_s}{J_g + J_s}
\]

(4.4)
The modal frequencies for the complex zero pair, $\omega_z$, and the complex pole pair, $\omega_p$, are:

$$\omega_z = \sqrt{\frac{K_g}{J_g}} \text{ rad/sec} \quad (4.5)$$

and

$$\omega_p = \sqrt{\frac{K_g}{J_p}} \text{ rad/sec} \quad (4.6)$$

Figure 4-3 shows the simplified block diagram for the flexible gimbal of figure 3-2 based on the mathematical development above.

![Simplified Block Diagram](image)

Figure 4-3

Simplified Block Diagram Showing the Flexible Body Modes for the Two Moving Bodies of the Three-Body Model of Fig 3-2

The figure shows the input and the four state variables for the angular position and rate of the gimbal and shaft bodies. Equations (4.5) and (4.6) give the pole and zero frequency locations $\omega_p, \omega_z$ respectively of the flexible mode. From the FEA we can place $\omega_p$ at $2\pi \ 363$ rad/sec. With equation (4.4), and from experience with flexible gimbal resonance’s, we can approximate the ratio $\omega_z$ to $\omega_p$ as:

$$\frac{\omega_z}{\omega_p} = \sqrt{\frac{J_p}{J_g}} \approx \sqrt{\frac{J_s}{J_g}} \approx \sqrt{\frac{1}{4}} = \frac{1}{2}$$

So the zero is about an octave below the pole, i.e. at 181Hz.

The transfer function, $\frac{y(s)}{u(s)}$, predicted by equation (4.3), is the open loop plant. We assume a damping ratio for the foamed composite gimbal as: $\zeta = 0.3$. So with $\omega_p = 2\pi \ 363 \text{ rad/sec}$ and $\omega_z = 2\pi \ 181 \text{ rad/sec}$; the normalized ($J_s = 1$) open loop transfer function becomes.

$$\frac{y(s)}{u(s)} = \frac{1 + \frac{2\zeta}{\omega_p} s + \frac{s^2}{2\zeta\omega_p}}{S^2 \left(1 + \frac{2\zeta}{\omega_z} s + \frac{s^2}{2\zeta\omega_z}\right)} = \frac{1 + 5.2 \times 10^{-4} s + 7.7 \times 10^{-7} s^2}{S^2 \left(1 + 2.6 \times 10^{-4} s + 1.9 \times 10^{-7} s^2\right)} \quad (4.7)$$
Figure 4-4 shows the expected open loop frequency response “rate plant” for the two-body flexible plant given by equation (4.7) with the low frequency form of \( \frac{1}{s} \) instead of \( \frac{1}{s^2} \).

4.2 Controller Design

Following is a simplified control system design based on the cascaded position and rate loop servo configuration shown in figure 4-5.

The initial design problem is to select the rate control compensator, \( G_{cr} \), to produce a wide bandwidth rate loop with adequate damping and stability margins. We select a very low frequency lead-integral (proportional-plus-integral) with a high frequency filter well beyond the resonance, i.e.:

\[
G_{cr} = K \frac{(1+s)}{s(1+\frac{s}{\omega_{filter}})}
\]

Figures 4-6a and 4-6b show the root locus and the closed loop Bode Plot for the compensated rate loop ignoring the high frequency filter break.
With the high gain, the root locus plot shows the complex pole migration is near the end of its travel and creates a very small residue dipole with the complex zero. The closed loop response of the rate loop shows the dipole effect near 200Hz but this has no effect on stability of the loop. The phase margin for the rate loop is about 90 degrees and the gain margin greater than 100dB.

The rate loop becomes the open loop plant for the position loop. We select a controller compensation similar to the rate loop. Ignoring the high frequency filter break, i.e.:

\[ G_{pr} = \frac{K(1+s)}{s} \]

Figures 4-7a and 4-7b show the root locus and the closed loop Bode plot for the compensated position loop.
As in the case of the rate loop, a small dipole results from the complex pole migration to the complex zero. The root locus plot shows the dominant closed loop second order poles with a damping ratio and natural frequency as of $\zeta = 0.7$ and $\omega_n = 6000 \text{rad/sec} \approx 1000 \text{Hz}$ respectively. This produces a dominant closed loop, second order transfer function as:

$$G_{\text{ClosedLoop}} \approx \frac{1}{1 + \frac{2\zeta}{\omega_n} s + \frac{s^2}{\omega_n^2}} = \frac{1}{1 + 2.3 \times 10^{-8} s + 2.8 \times 10^{-8} s^2} \quad (4.8)$$

The Closed loop response from Figure 4-7b shows that the gain is at –3dB near 200Hz indicating a bandwidth near that value. However, the behavior of the system will closely approximate the second-order transfer function in equation (4.8).
with a 1000Hz natural frequency. The phase and gain margins of the position loop are 75 degrees and >80dB respectively.

As shown in figures 4-1 and 4-5 we intend to use rate (and possibly, acceleration) feed-forward commands as part of the axis’ servo structure. By adding feed forward we produce two very important features in the servo system:
1) reduce the input/output phase lag of the position loop bandwidth making the servo appear to be a wider bandwidth system, and
2) permit the commands of rate and acceleration that will excite the axes mechanical gimbal system throughout the rigid body range.

Applying feed-forward changes the original position closed loop transfer function by adding a feed-forward lead term, i.e.:

\[ G_{PClosedLoop \ W} = G_{FF}(s) \cdot G_{PClosedLoop \ W/O} \]  

(4.9)

As an example, the rate feed-forward lead term is given by:

\[ G_{FF \ s \ rate} = 1 + \frac{s}{\omega_{np}} \]  

(4.10)

where \( \omega_{np} \) is the position loop natural (crossover) frequency. By adding the rate feed-forward command to the position loop we can compare the phase lag produced by the flight table servo at various frequencies for the composite flight table with that of a conventional flight table. Table 4-1 shows the comparisons of the composite flight table servo and the conventional table servo with a 30 Hz bandwidth and 0.7 damping ratio.

<table>
<thead>
<tr>
<th>Table 4-1</th>
<th>Phase Lag Comparisons between the 200Hz Bandwidth Composite Flight Table( Data from Figure 4-7b) and a 30 Hz Bandwidth Aluminum Flight Table</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase Lag at 10 Hz, Deg</td>
</tr>
<tr>
<td>Composite Flight Table <strong>without</strong> rate feed-forward</td>
<td>1</td>
</tr>
<tr>
<td>Conventional Flight Table <strong>without</strong> Rate Feed-Forward</td>
<td>30</td>
</tr>
<tr>
<td>Composite Flight Table <strong>with</strong> Rate Feed-Forward</td>
<td>&lt;1</td>
</tr>
<tr>
<td>Conventional Flight Table <strong>with</strong> Rate Feed-Forward</td>
<td>13</td>
</tr>
</tbody>
</table>

The table demonstrates the dynamic transparency of the wide bandwidth servo loop with the composite gimbaled system. There is practically no phase lag produced by the composite flight table below 150 Hz when using rate feed-forward.

**5. SUMMARY OF RESULTS**

We presented a design concept and analysis for a flight table consisting mostly of composite materials with cylindrical symmetry. The results demonstrate the feasibility of producing an “invisible” flight table.

Specifically, we produced a flight table design concept that achieved the following goals:
1. **Reduced costs** by using mostly commercial-off-the-shelf composite tubes for gimbals and shafts. The design exhibits low axis rotating inertias, low gimbal shear stresses, low localized bending areas and high axis E-modulus to density ratios.
2. **Reduced torque requirements** compared to conventional systems by a factor of 5. Because of the lower rotating inertias we can use electric brushless torque motors instead of hydraulic actuators.
3. **Increased location of lowest torsional modes** by almost a factor of 2.5 to 5.5 over the conventional flight table. The FEA analysis showed the lowest torsional resonant modes for the outer and middle axes to be of 363 and 835 Hz respectively.

4. **Produced negligible servo phase lag** as compared with the conventional system. The servo analysis for the outer axis demonstrated $<$ $\frac{1}{19}$ of phase lag from dc to 150 Hz.

### 6. FUTURE EFFORT

Acutronic believes that the composite flight table will find a demand at the HWIL laboratories in the U.S. and worldwide. However, before practical commercialization for this product begins, some questions must be answered first. These include:

1. Can the composite tubular design be manufactured inexpensively?
2. Can the composite tubular design produce the localized stress advantages over the aluminum rectangular design?
3. How well will the shaft mechanical interface eliminate the gimbal shear stresses and provide a rigid connection while undergoing high frequency, high dynamic motion?
4. Will the predictions of low rotating inertia and high torsional resonant modes hold true in a final, practical design?
5. Will the assumption of a large plant damping ratio (0.3 or greater) hold true in a practical design?
6. Will the lumped, 3-body model prove to be adequate enough to represent the more complex, distributed parameter system and still result in almost 200Hz bandwidth?
7. Can we validate the FEA model and actually produce the predicted resonant modes in the physical flight table?
8. Will the composite material properties (specifically $E$ and $\rho$) be stable over time?

The next logical step towards commercialization will be to build a “proof of principle” outer axis that will demonstrate the composite gimbal/flight table’s performance. The outer axis assembly will include the base, drive shafts, bearings, torque motors, Inductosyn feedback sensor, gimbal to shaft mechanical interface, simulated loads representing the inner and middle axes, and the control system. We propose to meet the following objectives during this next phase:

1. Complete the manufacturing and assembly drawings for the “proof of principle” outer axis
2. Construct the outer axis prototype, integrate the axis control system and conduct comprehensive testing including: static accuracy, dynamic performance and small signal frequency response. Compare test results with predictions.
3. Validate the Finite Element Model, the control system model and the composite to shaft mechanical interface design.
4. Investigate flexible mode control strategies using State Variable Feedback Control techniques.
5. Complete a final set of performance specifications, acceptance test procedures and an estimate of the manufacturability, cost and technical risks for the full flight table.

### 7. REFERENCES


